Exercise 91

The mass of part of a wire is $x \left(1 + \sqrt{x}\right)$ kilograms, where x is measured in meters from one end of the wire. Find the linear density of the wire when $x = 4$ m.

Solution

The linear density of a wire is the derivative of the mass with respect to the length.

$$\frac{dm}{dx} = \frac{d}{dx} \left[x \left(1 + \sqrt{x}\right) \right]$$

$$= \left[\frac{d}{dx}(x) \right] \left(1 + \sqrt{x}\right) + x \left[\frac{d}{dx} \left(1 + \sqrt{x}\right) \right]$$

$$= (1) \left(1 + \sqrt{x}\right) + x \left(\frac{1}{2} x^{-1/2} \right)$$

$$= \left(1 + \sqrt{x}\right) + \left(\frac{1}{2} x^{1/2} \right)$$

$$= 1 + \frac{3}{2} \sqrt{x}$$

Therefore, when $x = 4$ m, the linear density is

$$\left. \frac{dm}{dx} \right|_{x=4} = 1 + \frac{3}{2} \sqrt{4} = 4 \ \frac{\text{kg}}{\text{m}}.$$